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# English for Mathematics

ΕΚΔΟΣΕΙΣ ΖΗΤΗ

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# Preface

The motivation behind the writing of this book has been to provide a suitable ESP textbook for non-native speakers of English who are university or college students of mathematics, or the mathematical sciences. In this book we have attempted to address the following four points:

1. To provide a textbook that is a source of mathematical vocabulary and expressions useful for communicating mathematical ideas to an English speaking audience.

2. To provide a source of mathematical jargon and expressions that will be useful to university and college students undertaking studies in mathematics or the mathematical sciences.

3. To enable students on completion of the book, or relevant section(s) thereof, to confidently pursue their topic(s) of interest with relative ease.

4. To assist weak students of English to understand the contents of mathematical textbooks with relative ease.

Throughout the book, emphasis has been placed on learning the technical language of mathematics: as opposed to literally learning mathematics or English grammar. Consequently, the English used in this book has been kept at a level compatible with this aim. However, the passive voice has been freely employed since it is frequently used by native speakers. The book has been written in British English (as opposed to American English): hence rhythm, syntax, idiom, grammar and spelling conform to this system. In view of the previous comments, exercises involving calculations for the sake of mathematics or grammar alone are few. Wherever possible, exercises have been set that test the understanding of the English of the mathematical propositions, definitions and ideas: as well as the ability to express mathematical symbolism in simple English. A few words are now in order for both the instructor and the student.

Instructors using this book should place emphasis on teaching technical vocabulary and expressions. In our experience, most ESP students are primarily interested in being able to express themselves in their particular fields. Thus, to consume time explaining the subtleties of the underlying grammar would probably be, in our opinion, time lost: therefore a good balance should be sought between the necessary and the superfluous. Furthermore, classes should be streamed according to their ability in English: this will ultimately determine the style of teaching to be employed. Having dealt with the instructor let's now turn to the student.

This book is not country oriented, thus students will not find any multilingual glossaries in the appendices: the inclusion of such glossaries is not standard practise in EFL textbooks. In the event that problems arise concerning the translation of technical terms, then the student should consult his instructor or a good technical dictionary; this book is not a dictionary.

Students studying alone should place emphasis on learning the terms and the appropriate prepositions, or other standard expressions that may accompany them. If students meet with difficulties, then they should try to discover the equivalent of the given definitions or expressions in their native language. Beginners in English should try to master only simple terms and simple definitions. They should also try to find the equivalent of the given definitions or expressions in their native language.

In essence, this is neither a mathematical textbook nor a grammar book per se; the reader will not find the usual grammatical rules and exercises that may be found in a standard grammar book. Furthermore, the degree of rigour, which is found in some mathematical textbooks, has been relaxed in this book: since its purpose is not to teach mathematics. This does not mean that definitions are sloppy or wrong: it simply means that extremely rigorous definitions and applications are superfluous to the aim of the book. For example, it would be pedantic, in the extreme, to use relativistic mechanics to solve everyday kinematics problems just because relativity is more precise than Newtonian mechanics. The pedant has been warned!

On completion of the book, students will have enriched their mathematical vocabulary by over six hundred mathematical terms, and numerous expressions: a good foundation, we believe, for the further study of mathematical texts written in the English language.

ACKNOWLEDGEMENTS: Initially, Longman's mathematics textbook in the Nucleus series by D. Hall with T. Bowyer (Longman, 1991) had been used in the School of Mathematics at the University of the Aegean at Samos: unfortunately, this book was found lacking at this level of mathematics. Consequently, it was decided to write a book that would cover the needs of

the students more fully. However, we were guided, to some extent, by the approach taken in the Nucleus mathematics textbook and for this we are most grateful to the authors.

Additionally, we should like to express our gratitude to the authors and publishers of the following two textbooks, which greatly assisted us in the writing of this book: S. I. Grossman's Calculus (Academic Press, Inc., 1977) and J. Holt and D. Rohatyn's Logic (Schaum's Outline Series, 1988). Grossman's book was found to be most appropriate for sections 3, 4, 5, 7 and 8: the section in Grossman's book on quadric surfaces was especially useful. J. Holt and D. Rohatyn's book (which was found to be an excellent text for the section on introductory logic) provided the source for section 10, which has been written on similar lines to the approach taken in that book. It also should be noted that we have freely used problems from all of these books: both in their original forms and in modified forms.

Finally, but not least, we would like to thank the Department of Mathematics of the University of the Aegean for providing the resources for the creation of this book and Dr. A. Tsolomitis for all his support.

F. Evans    Athens, Greece
G. Danousis    Samos, Greece

*To my wife.*
*— F. E.*

*To my sons Mikhail and Nicholas.*
*— G. D.*

# Contents

# Chapter 1

# Geometry

KEYWORDS: *angle* (acute, adjacent, alternate, complementary, corresponding, depression, elevation, exterior, interior, negative, obtuse, positive, reflex, supplementary, vertical); *circle* (arc, centre, chord, circumference, circumscribed, concentric, diameter, inscribed, radius (pl. radii), secant); *curve*, *dimensions* (breadth, depth, height, length, width), gradient, *line* (bisector, horizontal, oblique, parallel, perpendicular, ray, segment, transversal, vertical); *plane* (coplanar); *point* (collinear, equidistant, intersection); *polygons* (decagon, heptagon, hexagon, irregular, nonagon, octagon, pentagon, regular); *quadrilaterals* (diagonal, parallelogram, rectangle, rhombus (pl. rhombi), square, trapezium (trapezoid)); *solids* (cone, cube, cuboid, cylinder, frustum, parallelepiped, pyramid, sphere, tetrahedron); *triangle* (altitude, base, congruent, equilateral, isosceles, right-angle, scalene, similar).

## 1.1 Lines, points, angles, and curves

### 1.1.1 Lines

All the definitions that follow relate to straight lines that are assumed to be in the same plane. Furthermore, throughout this chapter, the definitions have been presented in alphabetical order.

**Definition 1.1.1 (Bisector of a line segment)** *A line $AB$ that cuts/intersects another line segment $CD$ through its mid-point, i.e., the centre of the line segment, is said to bisect $CD$. In this case $AB$ is called the bisector of $CD$.*

Derivatives: to bisect (v); bisection (n).

**Definition 1.1.2 (Horizontal line)** *A line is said to be horizontal if the angle it makes with the ground is zero degrees.*

**Definition 1.1.3 (Oblique line)** *A line that is neither parallel nor perpedicular to another line is said to be an oblique line.*

Derivatives: obliquely (adv.).

**Definition 1.1.4 (Parallel line)** *Two line segments are said to be parallel to each other if they do not intersect: even when they are extended.*

**Definition 1.1.5 (Perpendicular line)** *Two lines are said to be perpendicular to each other if the angle between them is ninety degrees.*

**Definition 1.1.6 (Ray)** *The ray, $AC$, consists of the segment $\overline{AC}$ and all other points between $A$ and an arbitrary point $Q$ such that $C$ lies between $A$ and $Q$. The end point of the ray is $A$ —the point first named.*

**Definition 1.1.7 (Segment)** *The segment, $\overline{AC}$, of a line consists of the points $A$ and $C$ and all the points that are between them. $A$ and $C$ are called the endpoints of the segment.*

**Definition 1.1.8 (Transversal/Transverse line)** *A line that intersects two or more lines at different points is called a transversal/transverse line.*

Derivatives: to transverse (v).

**Definition 1.1.9 (Vertical line)** *A line is said to be vertical if the angle it makes with the ground is ninety degrees.*

### 1.1.2 Points

A point is a diagrammatic representation of a position in space. It is represented by a small dot, thus ".". We say that a point *lies* on a line.

**Definition 1.1.10 (Collinear)** *If two or more points lie on he same line, we say that the points are collinear.*

Derivatives: co-llinearity (n).

**Definition 1.1.11 (Equidistant)** *If a point is positioned at the same distance from two lines/points, we say that it is equidistant from those lines/points.*

Derivatives: equidistantly (adv.).

**Definition 1.1.12 (Point of intersection)** *If a line cuts across another line, then the point where the lines cut/intersect each other is called the point of intersection.*

Derivatives: to intersect (v).

### 1.1.3 Angles

**Definition 1.1.13 (Acute angle)** *An acute angle is an angle that is less than* $90^\circ$.

**Definition 1.1.14 (Adjacent angle)** *Two angles in a plane are called adjacent when they have a common vertex and a common side.*

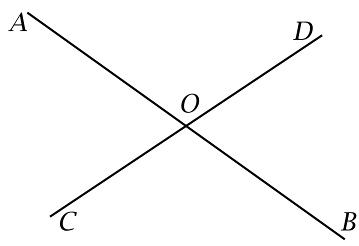


Figure 1.1.1: Adjacent angles

Referring to Figure 1.1.1: $\angle COB$ and $\angle BOD$ are called adjacent angles.

**Definition 1.1.15 (Alternate angles)** *Alternate angles are two non-adjacent angles on opposite sides of a transversal.*

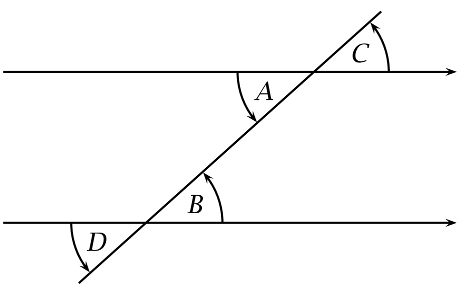


Figure 1.1.2: Alternate interior and exterior angles

Referring to Figure 1.1.2: $\angle A$ and $\angle B$ are called alternate interior angles. $\angle C$ and $\angle D$ are called alternate exterior angles.

**Definition 1.1.16 (Complementary angles)** *Complementary angles add up to* $90^\circ$*, e.g.,* $30^\circ + 60^\circ = 90^\circ$. $30^\circ$ *is the complement of* $60^\circ$ *and vice versa.*

Derivatives: to complement (v).

**Definition 1.1.17 (Corresponding angles)** *Two angles that are in corresponding positions relative to two lines are called corresponding angles.*

Derivatives: to correspond (v); correspondence (n).

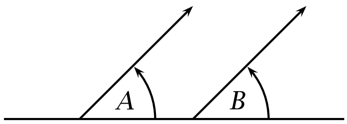


Figure 1.1.3: Corresponding angles

Referring to Figure 1.1.3: $\angle A$ and $\angle B$ are called corresponding angles.

**Definition 1.1.18 (Angle of elevation and angle of depression)** *Referring to Figure 1.1.4: $\angle A$ is called the angle of elevation (as measured from $R$ to $Q$). $\angle B$ is called the angle of depression (as measured form $Q$ to $R$). The line $RQ$ is called the line of sight.*

Derivatives: to depress, to elevate (v); depressed, elevated (adj.).

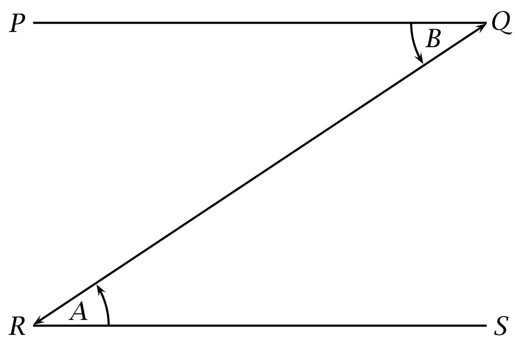


Figure 1.1.4: Angle of elevation and depression

**Definition 1.1.19 (Exterior angle)** *Exterior angles are angles that lie outside a shape or outside the space between two lines.*

**Definition 1.1.20 (Interior angle)** *Interior angles are angles that lie inside a shape or inside the space between two lines.*

**Definition 1.1.21 (Negative Angle)** *An angle is called a negative* (−) *angle if it is read in a clockwise direction.*

**Definition 1.1.22 (Obtuse angle)** *Obtuse angles are the angles that are less than* $180^\circ$ *but more than* $90^\circ$*.*

**Definition 1.1.23 (Positive Angle)** *An angle is called a positive* (+) *angle if it is read in an anti-clockwise direction.*

**Definition 1.1.24 (Reflex Angle)** *A reflex angle is an angle that is greater than* $180^\circ$*.*

**Definition 1.1.25 (Right Angle)** *A right angle is an angle that is equal to* (=) $90^\circ$*.*

**Definition 1.1.26 (Supplementary Angles)** *Supplementary angles add up to* $180^\circ$, *e.g.,* $120^\circ + 60^\circ = 180^\circ$*. The angle* $120^\circ$ *is called the supplement of* $60^\circ$ *and vice versa.*

Derivatives: to supplement (v).

**Definition 1.1.27 (Vertically opposite angles)** *Two angles are called vertically opposite when they lie on opposites sides of the point of intersection of two lines.*

Referring to Figure 1.1.5: $\angle AOD$ and $\angle COB$ are called vertically opposite angles as are $\angle AOC$ and $\angle DOB$.

**Exercise 1.1.1** Name each of the angles shown below:

| | | | | |
|---|---|---|---|---|
| (a) 60° | (b) 90° | (c) 175° | (d) 330° | (e) 40° |
| (f) 210° | (g) 181° | (h) 98° | (i) 359° | (j) 450° |

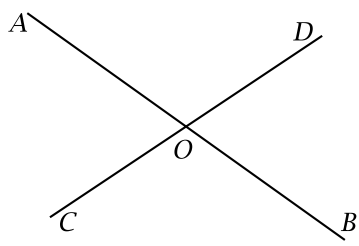


Figure 1.1.5: Vertically opposite angles

### 1.1.4 Curves

Lines that are not necessarily straight are called curves. However, the word curve is used more generally, and it may also be used to describe a straight line. We shall look at more complicated curves later. It should be noted that there are such things as good curves and bad curves; however, for the purpose of this book the student should not worry about these.

Curves usually have the following basic properties:

1. They have dimensions —here length.

   Derivatives: to lengthen (v); length (n); long (adj.).

2. They have gradients/slopes —this shows how steep a curve is at a particular point.

   Derivatives: to slope (v); sloped (adj.).

3. They can be represented by equations —mathematical representations of the curves (see later).

### 1.1.5 Dimensions

Let us suppose that the line $AC$ has a length of 5 cm.

Question: How long is the line $AC$?
Answer: It is 5 cm long.
Question: What is the length of the line $AC$?
Answer: The length of $AC$ is 5 cm.

*Note:* Strictly speaking we should say: "How long is the line *segment* $\overline{AC}$?" The student should not worry too much about this: it is a relatively minor point.

### 1.1.6 Gradient/slope

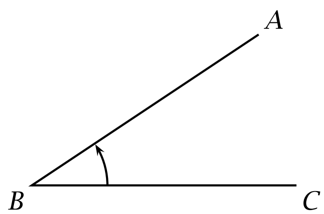


Figure 1.1.6: The slope of the line $BA$

Referring to Figure 1.1.6: the line $BA$ slopes at an angle $\angle ABC$ to the horizontal $BC$. The value of the gradient/slope of the line is equal to the tangent of $\angle ABC$.

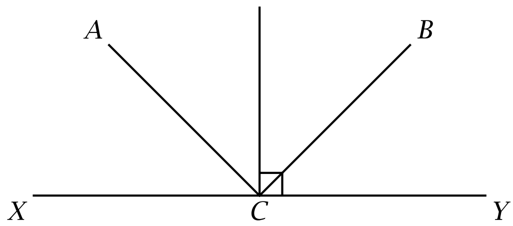


Figure 1.1.7: Negative and positive slopes

Referring to Figure 1.1.7: the line $CB$ has a positive slope/gradient. The line $CA$ has a negative slope/gradient.

**Exercise 1.1.2**

(i) Draw two lines that are parallel to each other.

(ii) Draw two lines that are perpendicular to each other.

(iii) Draw two lines that intersect at right angles to each other.

(iv) Draw a diagram that shows two vertically opposite angles.

(v) Draw a diagram that shows two corresponding angles.

(vi) Draw a diagram that shows two alternate interior angles.

(vii) Draw a diagram that shows two adjacent angles.

(viii) Draw a diagram that shows two alternate exterior angles.

(ix) Draw two lines such that one has a positive slope and the other a negative slope.

(x) Draw a line that has a zero gradient.

(xi) Draw a line that has an infinite slope.

(xii) Write down a reflex angle.

(xiii) Write down any two supplementary angles.

(xiv) Write down any two complementary angles.

**Exercise 1.1.3** Draw the geometrical constructions described below.

(i) The line segments $AB$ and $ED$ both have positive slopes. The line $ED$ bisects the line segment $AB$ at the point $C$ that lies on $AB$. The end point $D$, of the line $ED$, is connected by a straight line to the end point $B$ of the line $AB$. The line $GH$, which also has a positive slope, passes through the point $C$ and continues until it intersects the line $DB$ at the point $F$ on the line $DB$.

(ii) The horizontal line segment $CD$ bisects the vertical line segment $AB$ at the point $E$ that is on the line segment $AB$. Point $F$ is the mid-point of the line segment $EA$, and point $G$ is the mid-point of the line segment $CE$. The line segment $LM$ passes through the points $G$ and $F$. The line segment $HI$ is parallel to the line segment $AB$, and it passes through the point $K$ that is the mid-point of the line segment $ED$.

(iii) The line segments $AB$ and $CD$ are parallel to each other. The transverse line segment $HL$, which has a negative slope, bisects the line segments $AB$ and $CD$ at their mid points $M$ and $N$, respectively. Point $P$, the mid-point of the line segment $HL$, lies between the line segments $AB$ and $CD$. The line segment $HL$ passes through the point $Q$. A line segment has been drawn through the point $Q$ to the point $R$ that is the mid-point of the line segment $ND$.

(iv) The line segment $AC$ is a horizontal line segment. The points $B$ and $D$ are positioned equidistantly above and below the mid-point $E$ of the line segment $AC$: the point $B$ lies above the line segment and the point $D$ lies below it. The points $A$, $B$, $C$ and $D$ are connected in turn to each other by straight line segments: terminating at the point $A$.

(v) The line segments $AB$ and $CD$ are horizontal to the line segment $EF$ that is the perpendicular bisector of the line segments $AB$ and $CD$: intersecting these line segments at the points $M$ and $N$, respectively. From the point $C$ a line segment passes through the line segment $MN$ to the point $B$. From the point $D$ a line segment passes through the line segment $MN$ to the point $A$.

## 1.2 Plane figures (shapes)

In this section we shall look at four closed shapes: triangles, quadrilaterals, polygons, and circles. These shapes are collectively called plane shapes because they lie in a plane, i.e., a flat surface. Points and lines that lie in the same plane are said to be coplanar.

### 1.2.1 Triangles

A triangle is a three-sided plane shape, and the sum of its interior angles is 180°. The principal parts of a triangle are shown in Figure 1.2.1. Derivatives: triangular (adj.).

**Exercise 1.2.1** Using Figure 1.2.1 write definitions for the following parts of a triangle: altitude, median and base.

**Definition 1.2.1 (Congruent Triangles)** *Triangles that have the same size and geometric shape are called congruent triangles.*

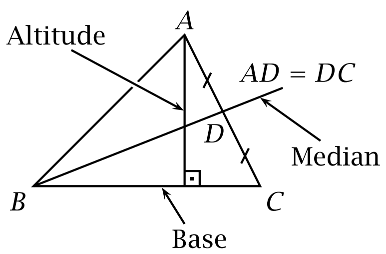


Figure 1.2.1: A triangle

Referring to Figure 1.2.2: triangle $ABC$ is congruent (≅) to triangle $DEF$ since their corresponding sides and angles are equal, i.e., $AC = DF$, $BC = EF$, $BA = ED$, and $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$. Derivatives: congruence (n).

**Definition 1.2.2 (Equilateral Triangle)** *An equilateral triangle is a triangle that has all its sides the same length.*

**Definition 1.2.3 (Isosceles Triangle)** *An isosceles triangle is a triangle that has any two of its sides the same length.*

**Definition 1.2.4 (Right-angled Triangle)** *A triangle that has one of its angles equal to ninety degrees is called a right-angled triangle.*

**Definition 1.2.5 (Scalene Triangle)** *A scalene triangle is a triangle none of the sides of which are the same length.*

**Definition 1.2.6 (Similar Triangles)** *Triangles that have their corresponding angles of equal size but which have their corresponding sides of unequal length are called similar triangles.*

Referring to Figure 1.2.3: triangle $ABC$ may be split into two triangles. We say that triangle $ABC$ is similar ($\sim$) to triangle $DBE$ since their corresponding angles are equal, i.e., $\angle A = \angle D$, $\angle B = \angle B$ and $\angle C = \angle E$.

Derivatives: similarity (n).

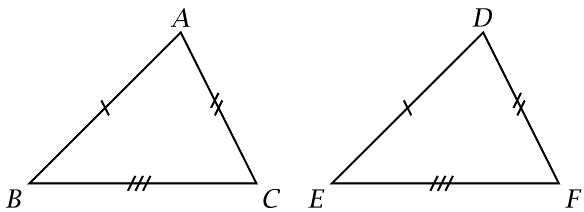


Figure 1.2.2: Triangle $ABC$ is congruent to triangle $DEF$

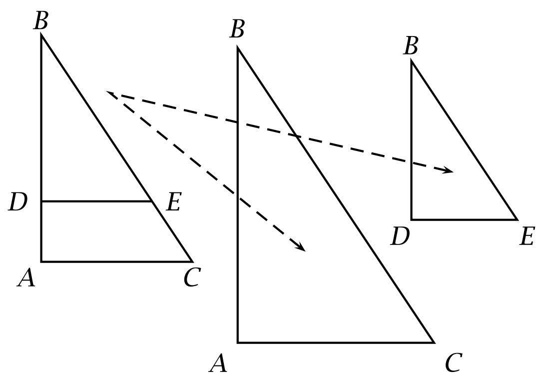


Figure 1.2.3: Similar triangles

### 1.2.2 Quadrilaterals

A quadrilateral is a four-sided plane shape. Five quadrilaterals are described below.

**Definition 1.2.7 (Parallelogram)** *A parallelogram is a quadrilateral that has its opposite sides parallel and equal in length, but the interior angles are not right angles.*

**Definition 1.2.8 (Rectangle)** *A rectangle is a quadrilateral that has its opposite sides parallel and equal in length, and the interior angles are right angles.*

**Definition 1.2.9 (Rhombus)** *A rhombus is a parallelogram that has all its sides the same length. The opposite sides are parallel, but the interior angles are not right angles.*

**Definition 1.2.10 (Square)** *A square is a quadrilateral that has all its sides the same length: the opposite sides are parallel, and the interior angles are right angles.*

**Definition 1.2.11 (Trapezium/Trapezoid)** *A trapezium (or trapezoid) is a quadrilateral with two parallel sides of different lengths. The parallel sides are called the bases of the trapezium. The interior angles are not right angles.*

**Exercise 1.2.2** Write down one or two properties for each quadrilateral: do not includ those properties that have already been given. *Hint:* think about the properties of the interior angles and the diagonals (a diagonal line is a straight line, not being aside, drawn from one vertex to another).

### 1.2.3 Polygons

A polygon is a multi-sided plane shape.
Derivatives: polygonal (adj.).

**Definition 1.2.12 (Regular and Irregular polygons)** *If a polygon is called a regular polygon, then it is both equilateral (all its sides have the same length) and equiangular (all its angles have the same size); otherwise it is called an irregular polygon.*

The following are some examples of how to name polygons:

| | |
|---|---|
| Pentagon | 5 sides |
| Hexagon | 6 sides |
| Heptagon | 7 sides |

| | |
|---|---|
| Octagon | 8 sides |
| Nonagaon | 9 sides |
| Decagon | 10 sides |

Notice how Greek numbers have been used in naming the polygons.

**Exercise 1.2.3** Write a numbered list of instructions explaining how to draw a regular polygon with a ruler and protractor.

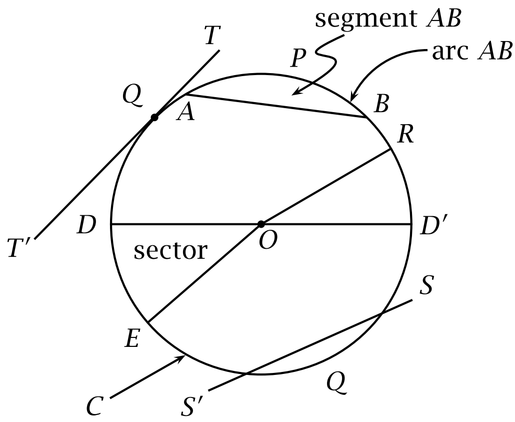


Figure 1.2.4: A circle

### 1.2.4 Circles

We shall now look at circles and some of the important terms that have to do with them.

Derivatives: to circle (v); circular (adj.).

Referring to Figure 1.2.4: $TT'$ is the tangent to the circle at $Q$; $AB$ is a chord; $OR$ is the radius of the circle; $DD'$ is the diameter of the circle; $O$ is the centre of the circle; $C$ is the circumference of the circle which equals $2\pi r$; $SS'$ is a secant (a line segment that intersects the circle at two points) and $ODE$ is a sector of the circle.

*Note:* The Greek letter $\pi$ is pronounced the same as the word pie.

In Figure 1.2.4 the chord $AB$ divides the circle into two arcs and two segments: the minor arc $APB$ and minor segment $APB$ (where the latter is the area between the chord $AB$ and the arc $APB$), and the major arc $AQB$ and the major segment $AQB$ which are on the opposite side of the chord $AB$. Notice that the major segment is the area between the chord $AB$ and the arc $AQB$.

As you may have guessed, the naming corresponds with the size of the arcs and the segments. Arcs and segments that are preceded by the word minor are smaller than those arcs and segments that are preceded by the word major.

The definitions that follow refer to circles.

**Definition 1.2.13 (Concentric Circles)** *Two or more circles that have the same centre are called concentric circles.*

Derivatives: concentricity (n).

**Definition 1.2.14 (Circumscribed and Inscribed)** *A circle is said to be circumscribed about a square if each corner of the square lies on the circumference of the circle. On the other hand, a circle is said to be inscribed in a square if the sides of the square are tangent to the circumference of the circle.*

**Definition 1.2.15 (Line of Centres)** *A line that passes through the centres of two or more circles is called the line of centres.*

Derivatives: to centralise (v); central (adj.).

**Exercise 1.2.4** Draw the following geometrical constructions.

(i) A circle with centre $O$ is inscribed in the square $ABCD$. The sides $AB$ and $DC$ of the square are horizontal. From the centre $O$ of the circle, a line segment, $OP$, is drawn so that it bisects the side $AB$ of the square.

(ii) The tangents at the points $P$ and $Q$, respectively, on the circumference of the circle centre $O$ are extended to meet at the point $T$. The radius, $OR$, of the circle is also extended to meet the tangents at $T$.

(iii) A circle with centre $O$ is circumscribed about the square $ABCD$, and a concentric circle is inscribed in the same square.

(iv) The line segment $AB$ is tangent to two circles, centres $O$ and $O'$ respectively, at the point $P$. The line of centres of the circles, $ST$, passes through $P$.

(v) Two circles intersect at the points $P$ and $P'$. A line segment is drawn through the points $P$ and $P'$ that intersects the line of centres $ST$, of the two circles, at the point $Q$.

### 1.2.5 Properties of plane figures (shapes)

Plane shapes have the following properties:

1. Dimension —here length and width/breadth.

   Derivatives: to widen, to broaden, to lengthen (v); long, wide, broad (adj.).

2. Perimeter —the distance around the shape.

3. Plane area —the amount of space that the shape covers.

**Dimensions** Consider a rectangle of length 20 cm and width 8 cm. The following are some typical questions:

Question: What is the length of the rectangle?
Answer: The rectangle is 20 cm long.

OR

The length of the rectangle is 20 cm.

Question: What is the width/breadth of the rectangle?
Answer: The width/breadth of the rectangle is 8 cm.

OR

The rectangle is of width/breadth 8 cm.

Question: What is the length of the perimeter of the rectangle?
Answer: The perimeter of the rectangle is 56 cm long.

OR

The perimeter of the rectangle is 56 cm in length.
(i.e., $20 + 8 + 20 + 8 = 56$ cm)

Question: What is the area of the rectangle?
Answer: The area of the rectangle is 160 $\text{cm}^2$ (which is read as: 160 square centimetres).

Now consider a circle of radius 8 cm.
Question: What is the length of the radius of the circle?
Answer: The radius is 8 cm long.

OR

The length of the radius is 8 cm.

Question: What is the circumference of the circle?
Answer: The circumference of the circle is $2\pi r = 16\pi$.
*Note:* With a circle we use the word circumference instead of perimeter.

**Exercise 1.2.5** Write similar questions and answers for the following shapes:

(i) A circle of radius 9 cm.

(ii) A rectangle of length 18 cm and width 6 cm.

## 1.3 Three dimensional, 3D, figures (shapes)

Eight 3D shapes are dealt with in this section: the cone, the cube, the cuboid, the cylinder, the frustum, (pl. frusta), the parallelepiped, the pyramid/tetrahedron, and the sphere. A 3D shape is either solid or hollow: if it is solid, then interior of the shape is full; otherwise it is called hollow and it is empty.

### 1.3.1 The Cone

Figure 1.3.1 is a diagram of a right circular cone.
Derivatives: conical (adj.).

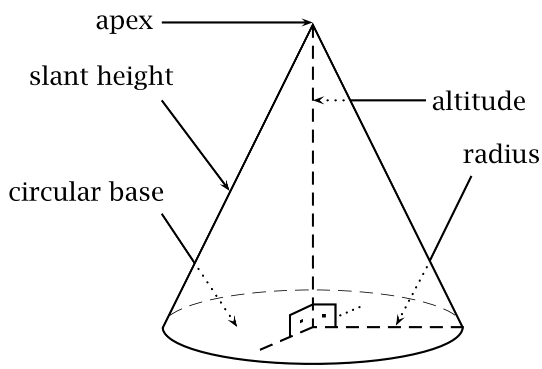


Figure 1.3.1: A cone

### 1.3.2 The Cube

The cube has all its edges the same length, and its opposite edges are parallel. The interior angles are right angles. If any two of its dimensions are different, then the shape is called a cuboid: both have six lateral faces, twelve edges and eight corners/vertices (singular vertex).

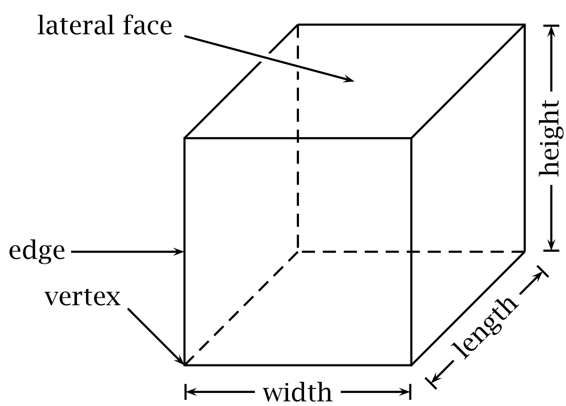


Figure 1.3.2: A cube

### 1.3.3 The Cylinder

Figure 1.3.3 and 1.3.4 show two cylinders: a solid cylinder and a hollow cylinder, respectively. The sides are parallel and at right angles to the circular base.

In the case of the hollow cylinder: if $O$ is the centre of the circular base, then $OA$ is called the outer radius and $OB$ is called the inner radius. Thus $OA$ minus $OB$ equals the thickness of the cylinder. E.g., if $OA = 10\,\text{cm}$ and $OB = 8\,\text{cm}$, then the thickness of the cylinder is $10 - 8 = 2\,\text{cm}$.

Derivatives: cylindrical (adj.).

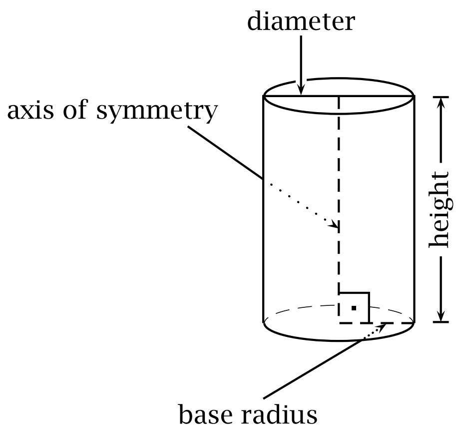


Figure 1.3.3: A cylinder

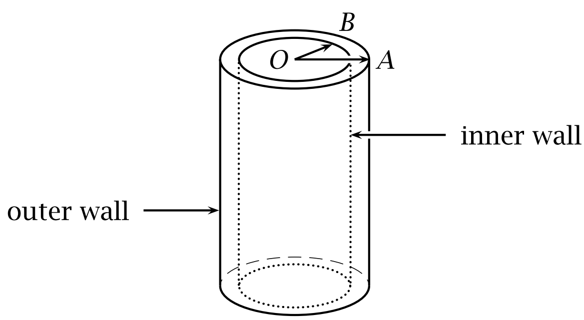


Figure 1.3.4: A hollow cylinder

The following are typical questions:
Question: What is the thickness of the cylinder?
Answer: The cylinder is 2 cm thick.

OR

The cylinder has a thickness of 2 cm.

### 1.3.4 The Frustum

Figure 1.3.5 shows how the frustum of a cone is formed from a cone.

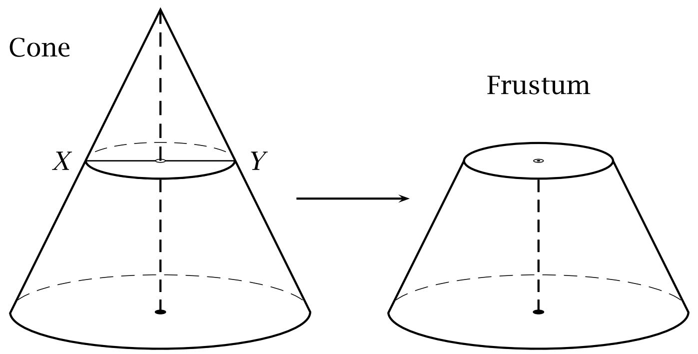


Figure 1.3.5: Frustum of a cone

**Exercise 1.3.1** Sketch the frustum of a square pyramid.

### 1.3.5 The Parallelepiped

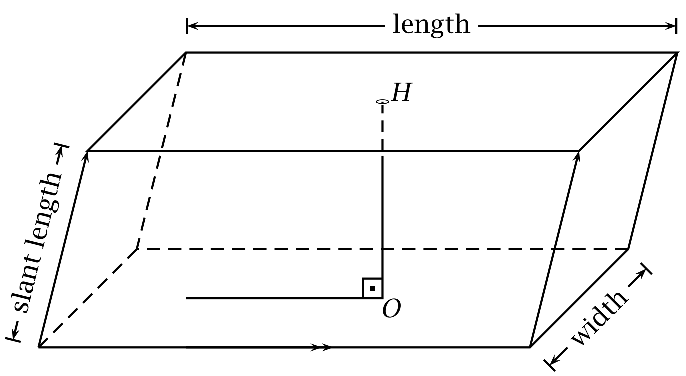


Figure 1.3.6: A parallelepiped

Opposite parallel sides have the same length. The angles are not right angles: in the case that all the angles are right angles —the parallelepiped becomes a cuboid. In Figure 1.3.6, the distance $OH$ is the perpendicular height.

### 1.3.6 The Pyramid/Tetrahedron

Figure 1.3.7 shows a rectangular based tetrahedron. Notice the similarities between it and the right-circular cone in Figure 1.3.1.

Derivatives: tetrahedral (adj.).

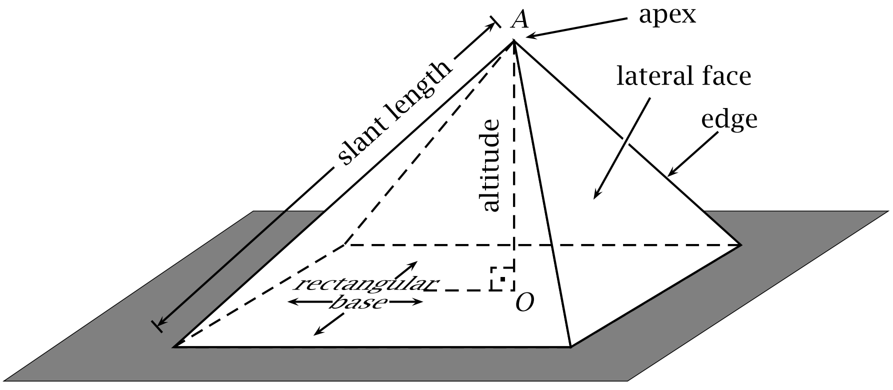


Figure 1.3.7: A pyramid

**Exercise 1.3.2** Sketch a square based pyramid and label the following parts: the base, the altitude, the apex, the slant length, and one lateral face. How many faces, edges, and vertices has it got?

### 1.3.7 The Sphere

The sphere is the 3D analogue of the circle.

**Exercise 1.3.3** Sketch a sphere and label the following parts: the centre, the radius, the diameter, and the circumference.

Solid shapes have the following basic properties:

1. Dimension —here length, width/breadth, height, and depth.

   Derivatives: to deepen, to heighten (v); deep, high (adj.).

2. Lateral surface area —this is a measure of the sum of the areas of the solid's faces.
3. Cross-sectional area —this is the surface area of a slice of the solid that has been cut perpendicular to an axis of symmetry of the solid.
4. Volume —this is a measure of the space that a solid occupies.

### 1.3.8 Dimensions

All the dimensions except for depth have been looked at. Depth is like height, but whereas height goes upwards —depth goes downwards. A typical question might be:

Question: How deep is the swimming pool?

OR

What is the depth of the swimming pool?

Answer: It is $4m$ deep.

OR

It has a depth of $4m$.

Depth may also be measured at two very important places: the shallow end and the deep end of a solid. For example, the depth of the water at the shallow end of a swimming pool may only be 1 metre, but the depth of the water at the deep end of the pool may be as much as 4 metres.

### 1.3.9 Lateral surface area

To find the lateral surface area of a solid we often need to know a formula (pl. formulae or formulas) in order to do so. For instance, a cube of side length x cm has a lateral surface area given by $6x^2$. If a value of 5 cm is entered into this formula, then the lateral surface area is 150 $\text{cm}^2$ (which is read as: 150 square centimetres).

### 1.3.10 Cross-sectional area

If the cuboid in Figure 1.3.8 is cut through parallel to the face $WXYZ$ and perpendicular to the base, then the resulting rectangle $ABCD$ is said to be the cross-sectional area. In this case the area is 48 $\text{cm}^2$.

### 1.3.11 Volume

If we consider the cuboid shown in Figure 1.3.8, then the volume of the cuboid is 576 $\text{cm}^3$ (which is read as: 576 cubic centimetres or 576 centimetres cubed).

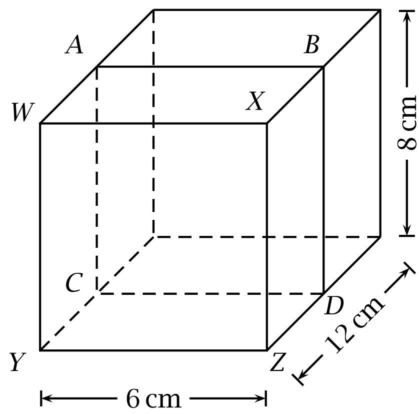


Figure 1.3.8: A cuboid

**Exercise 1.3.4** For the solids below, write questions and answers for each of the following items of interest: length, width, height/depth, lateral surface area, cross-sectional area, and volume. Some of the items may not be applicable to some of the shapes.

(i) A sphere of radius 4 cm.

(ii) A cylinder of height 20 cm and base diameter of 12 cm.

(iii) A right circular cone of height 6 cm, radius 8 cm, and slant length of $10cm$.

## 1.4 Listening exercises

**Exercise 1.4.1** Place a number against the shapes in the order that you hear them being described. The first one has been done for you.

| SHAPE | ORDER |
|---|---|
| sphere | |
| cube | |
| parallelepiped | |
| circular cone | |
| solid cylinder | |

| SHAPE | ORDER |
|---|---|
| hollow cylinder | |
| rhombus | |
| square pyramid | |
| trapezium | |
| circle | 1 |

**Exercise 1.4.2** You will hear five geometrical constructions (*A*, *B*, *C*, *D* and *E*) being described: for each description draw and label the diagram as instructed.

**Exercise 1.4.3** You will hear ten statements. For each statement decide whether it is true or false and write T or F, accordingly, next to each number.

| | | | | |
|---|---|---|---|---|
| 1. | 2. | 3. | 4. | 5. |
| 6. | 7. | 8. | 9. | 10. |